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1. AGENCY USE ONLY (Leave blank)		2. REPORT DATE March 1993		3. REPORT TYPE AND DATE Professional Paper	
4. TITLE AND SUBTITLE THE EFFICIENT CALCULATION AND DISPLAY OF DISPERSION CURVES FOR A THIN CYLINDRICAL SHELL IMMERSSED IN A FLUID				5. FUNDING NUMBERS PR: SW14 PE: 0602314N WU: DN308107	
6. AUTHOR(S) H. A. Schenck					
7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES) Naval Command, Control and Ocean Surveillance Center (NCCOSC) RDT&E Division San Diego, CA 92152-5001				8. PERFORMING ORGANIZATION REPORT NUMBER	
9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES) Naval Command, Control and Ocean Surveillance Center (NCCOSC) RDT&E Division San Diego, CA 92152-5001				10. SPONSORING/MONITORING AGENCY REPORT NUMBER	
11. SUPPLEMENTARY NOTES					
12a. DISTRIBUTION/AVAILABILITY STATEMENT Approved for public release; distribution is unlimited.				12b. DISTRIBUTION CODE	
13. ABSTRACT (Maximum 200 words) Much of the extensive literature on shell theory centers around the computation of dispersion curves, usually thought of as the roots of a determinant D which relates the angular frequency ω to the wave numbers for free waves traveling on the shell. For a cylindrical shell, the free wave paths are actually helical curves on the surface of the shell, having both a circumferential and an axial wave number. In this case, it is useful to indicate explicitly that the dispersion relation is a function of three variables, i.e., $D(\omega, m, n) = 0$, where m is the number of half wavelengths of a wave traveling in the axial direction, and n is the number of full wavelengths of a wave traveling around the shell. We shall refer to this function $D(\omega, m, n)$ as the <i>dispersion volume density</i> , since it is a function of three variables. Normally, one sees various two-dimensional displays of the dispersion relation, which are created by plotting any two of these variables against each other, while holding the third variable fixed.					
<div style="text-align: right;">93-11429</div> <div style="text-align: right;"> </div> <p>Published in <i>Proceedings</i>, 14th International Congress on Acoustics, Vol. 1, 3-10 Sep 1992, paper B8-2.</p>					
14. SUBJECT TERMS target strength active surveillance martsam				15. NUMBER OF PAGES	
				16. PRICE CODE	
17. SECURITY CLASSIFICATION OF REPORT UNCLASSIFIED	18. SECURITY CLASSIFICATION OF THIS PAGE UNCLASSIFIED	19. SECURITY CLASSIFICATION OF ABSTRACT UNCLASSIFIED	20. LIMITATION OF ABSTRACT SAME AS REPORT		

UNCLASSIFIED

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B8-2

THE EFFICIENT CALCULATION AND DISPLAY OF DISPERSION CURVES FOR A THIN CYLINDRICAL SHELL IMMERSSED IN A FLUID

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Much of the extensive literature on shell theory centers around the computation of dispersion curves, usually thought of as the roots of a determinant D which relates the angular frequency ω to the wave numbers for free waves traveling on the shell. For a cylindrical shell, the free wave paths are actually helical curves on the surface of the shell, having both a circumferential and an axial wave number. In this case, it is useful to indicate explicitly that the dispersion relation is a function of three variables, i.e., $D(\omega, m, n) = 0$, where m is the number of half wavelengths of a wave traveling in the axial direction, and n is the number of full wavelengths of a wave traveling around the shell. We shall refer to this function $D(\omega, m, n)$ as the *dispersion volume density*, since it is a function of three variables. Normally, one sees various two-dimensional displays of the dispersion relation, which are created by plotting any two of these variables against each other, while holding the third variable fixed, as illustrated in Fig. 1

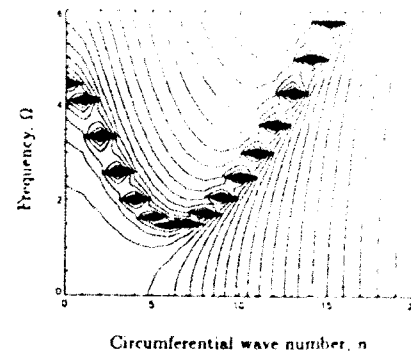
Dispersion relations for a fluid-loaded cylindrical shell have been developed previously by many authors. However, as pointed out in a lengthy article by Scott [1], some of these previously published works (e.g., [2] and [3]) made the mistake of searching for roots of the *real* part of the dispersion equations, which led to the computations of spurious non-physical branches of the dispersion curves. Although Scott has convincingly made this point, his actual computation procedure is cumbersome. In fact, it is difficult to determine the complex roots of such dispersion relationships by any root-following method. In the present work, the complications of calculating and following these complex roots have been simplified by using a regular computational grid for each parameter of the dispersion function, and then making the root loci evident by the appropriate use of graphics to display what we term the *dispersion volume density*.

The purpose of this paper is to describe and illustrate an efficient procedure for calculating and displaying the dispersion curves. We will base the results on Flügge shell theory, which has some advantages for air-backed thin steel shells immersed in water. Only the critical steps and results in the derivation of the dispersion relationships will be given here.

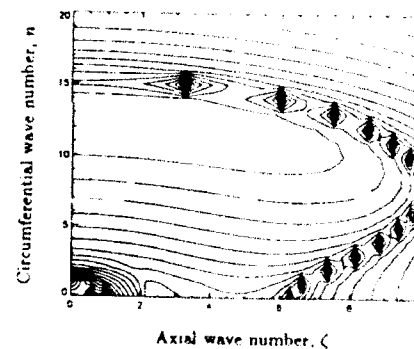
The displacements and forces are expanded in a combined Fourier series and transform to allow for completely general motions due to asymmetric forcing functions. With a time-harmonic factor of $e^{i\omega t}$ suppressed, the displacements u^r, u^θ, u^z of the mid-surface of a straight-sided cylindrical shell of mean radius a and thickness h are a function of z and ϕ only. In general, the forcing function F will also have three degrees of freedom, but F will be identically zero for a shell in *vacuo*, and $F^z = F^\theta = 0$ when the only force causing or resisting motion of the shell is $p(a, \phi, z)$, the total (incident plus scattered) pressure on the shell due to an incident acoustic wave. The shell material will be characterized by three parameters: Poisson's ratio ν , Young's modulus E , and the density ρ_s ; the fluid by a density ρ_f and a speed of sound c_f . Other (non-dimensional) parameters describing the solution are

$$\begin{aligned} \text{shell thickness/mean radius} &= h/a \\ \text{length/mean radius} &= l/a \\ \text{thickness parameter } \beta^2 &= h^2/12a^2 \\ s &= z/a \\ \Omega &= k_p = \omega/c_p \\ c_p^2 &= E/\rho_s(1-\nu^2) \\ \gamma &= r/a \end{aligned}$$

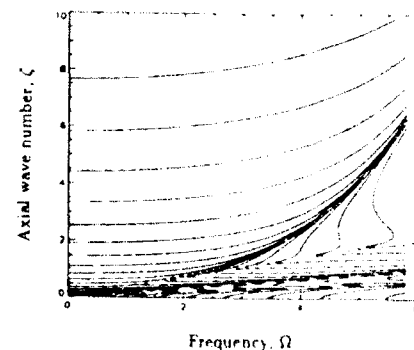
We assume that $u^r(\phi, s)$ can be expanded in a Fourier series in ϕ , i.e.,



(c) Frequency Ω vs. harmonic n for fixed ζ



(b) 'Figure 8' pattern, n vs. ζ for fixed Ω



(a) Conventional dispersion plot, ζ vs. Ω for fixed n

Fig. 1 Cuts along the principal axes of the *dispersion volume density* for a thin cylindrical shell in water

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$$u^r(\phi, s) = \sum_{n=-\infty}^{\infty} u_n^r(s) e^{-in\phi} \quad (1)$$

and that the axial variation of $u_n^r(s)$ can be expressed as a Fourier transformation

$$u_n^r(s) = \int_{-\infty}^{\infty} U_n^r(\zeta) e^{-i\zeta s} d\zeta, \quad (2)$$

where the transform variable ζ is a non-dimensional axial wavenumber. Similar expansions are assumed for $u^s(\phi, s)$ and $u^r(\phi, s)$. Because of the orthogonality properties of the Fourier expansions, the application of the operators to the expansions for the displacement results in an infinite set of matrix equations for each ζ , one for each harmonic order n . The differential operators all become matrix operators, resulting in a set of coupled linear equations. We also represent the acoustic pressure in a double Fourier expansion which allows us to express the forcing term as a function of $U_n^s(\zeta)$, the double Fourier expansion of the radial displacement. This permits us to move the forcing term in the transformed equations over to the left-hand side and write a completely homogeneous matrix equation for the Fourier components of shell displacement:

$$KU = \begin{bmatrix} K_{00} & K_{01} & K_{02} \\ K_{10} & K_{11} & K_{12} \\ K_{20} & K_{21} & K_{22} \end{bmatrix} \begin{bmatrix} U_n^s(\zeta) \\ U_n^r(\zeta) \\ U_n^s(\zeta) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}. \quad (3)$$

The components of K in equation (3) are given by

$$K_{00} = -\zeta^2 - \frac{(1-\nu)}{2}n^2 + (k_p a)^2 - \beta^2 \left[\frac{(1-\nu)}{2}n^2 \right] \quad (4)$$

$$K_{01} = K_{10} = -n\zeta \frac{(1+\nu)}{2} \quad (5)$$

$$K_{02} = K_{20} = -i\zeta\nu - i\zeta^3 + i\zeta n^2 \frac{(1-\nu)}{2} \quad (6)$$

$$K_{11} = -\frac{(1-\nu)}{2}\zeta^2 - n^2 + (k_p a)^2 - \frac{3(1-\nu)}{2}\zeta^2 \quad (7)$$

$$K_{12} = K_{21} = -in - in\zeta^2 \frac{(3-\nu)}{2} \quad (8)$$

$$K_{22} = 1 + \beta^2(\zeta^2 + n^2)^2 - (k_p a)^2 + \beta^2(1-2n^2) - \frac{\Omega^2 m_n(\zeta)}{\rho_s h} + i\Omega \frac{a r_n(\zeta)}{\rho_s h c_p}, \quad (9)$$

in which the additional terms to account for the resistance and reactance components of the fluid loading are

$$r_n(\zeta) = \omega \rho_f a \operatorname{Im} \left[\frac{H_n^{(2)}(y)}{y H_n^{(2)\prime}(y)} \right], \quad (10)$$

$$m_n(\zeta) = -\rho_f a \operatorname{Re} \left[\frac{H_n^{(2)}(y)}{y H_n^{(2)\prime}(y)} \right], \quad (11)$$

and $y = \sqrt{(ka)^2 - \zeta^2}$ for frequencies such that $ka \geq \zeta$ ($k \geq k_s$). When $ka < \zeta$ ($k < k_s$, the so-called evanescent region),

$$r_n(\zeta) = 0 \quad \text{and} \quad m_n(\zeta) = -\rho_f a \frac{K_n(x)}{x K_n'(x)}, \quad (12)$$

$$\text{where } x = \sqrt{\zeta^2 - (ka)^2}.$$

Equation (3) can have a non-trivial solution whenever its determinant vanishes, which can occur for particular combinations of frequency Ω , axial wavenumber ζ , and circumferential wavenumber n when there is no fluid loading. These combinations represent the dispersion relations for this shell in *vacuo*. Fluid loading introduces (acoustic) damping or loss. Even in this case, the absolute value of the determinant can become very small, and the resultant forced solution can be very large for particular combinations of Ω , ζ , and n . Thus minima of the absolute value of the determinant of K will define the dispersion relations for the fluid-loaded case.

The simplified computational procedure is to compute the coefficients of the matrix K and then evaluate the determinant of K at regular increments in the principal variables which are the non-dimensional frequency Ω , axial wavenumber ζ , and circumferential wavenumber n . The power of the method lies in the fact that modern graphical workstations with appropriate visualization software can not only perform these calculations but can quickly display the results in a manner which allows one to understand the dispersion relations in a global sense (over the entire meaningful range of Ω , ζ , and n). The actual program code was written in an efficient high level mathematical language.

Once the *dispersion volume density* is calculated for an appropriate range and density of Ω , ζ , and n , the results can be displayed in a variety of ways. One of the simplest and most effective means is compute the logarithm of the reciprocal of the absolute value of D , and then create a smooth interpolated color image of the results. If the color table is properly chosen, this has the effect of showing the root loci as lines whose brightness indicates the nearness to zero and whose width is indicative of width of the null and of the resolution in the appropriate parameters (Ω , ζ , and n). If calculated in this way, it is also possible to animate the display of these color images, which enhances the understanding of how the location of the modes change with changes in various parameters. All of these ideas will be illustrated in the presentation of this paper, but cannot be adequately shown in this brief black and white hardcopy.

In summary, the present work confirms the earlier work of Scott as to the correct dispersion loci, but provides a simpler and more easily interpreted method of calculating and displaying the *dispersion volume density* over the entire relevant range of frequency and wavenumbers.

This work was supported by the Naval Research Laboratory and the Office of Naval Technology. The suggestions and critique of the mathematics and computational procedures by George W. Benthien is gratefully acknowledged.

1. J.F.M. Scott, *The free modes of propagation of an infinite fluid-loaded thin cylindrical shell*, J. Sound Vib. 125 241-280 (1988)
2. M. C. Junger, and J.M. Garrellick, *Multiple modal resonances of thin cylindrical shells vibrating in an acoustic medium*, J. Acoust. Soc. Am. 75 1380-1382 (1984)
3. V. V. Musychenko and S.A. Rybak, *Low-frequency resonance scattering of sound by finite cylindrical shells (review)*, Sov. Phys. Acoust. 34(4) 325-333 (1988)

14th International Congress on Acoustics
ERRATA SHEET
Paper B8-2

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In the original submission of this paper, a factor of β^2 was omitted from several
equations. Equations (6)-(8) of Paper B8-2 should read as follows:

$$K_{02} = K_{20} = -i\zeta\nu - i\beta^2\zeta^3 + i\zeta n^2 \frac{(1-\nu)}{2} \beta^2 \quad (6)$$

$$K_{11} = -\frac{(1-\nu)}{2}\zeta^2 - n^2 + (k_p a)^2 - \frac{3(1-\nu)}{2}\zeta^2 \beta^2 \quad (7)$$

$$K_{12} = K_{21} = -in - in\zeta^2 \frac{(3-\nu)}{2} \beta^2 \quad (8)$$